

Answer Key :- SET-A

(1)

SECTION-A :-

- 1)  $5\pi/6$
- 2)  $-\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$
- 3)  $x = 3$
- 4)  $\frac{3}{7}$  (OR)  $P(E) = \frac{13}{52}, P(F) = \frac{4}{52} \text{ & } P(E \cap F) = \frac{1}{52}$   
Since  $P(E) \cdot P(F) = P(E \cap F)$   $\therefore E \text{ & } F$  are independent.

SECTION-B :-

- 5) Let  $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Solving the above, we get,  $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

- 6) d.r's of the 2 lines are  $5\lambda+2, -5, 1$  &  $1, 2\lambda, 3$ .  
Since they are  $1^r$ ,  $(5\lambda+2)1 + (-5)2\lambda + 1(3) = 0 \Rightarrow \lambda = 1$ .

- 7)  $x = -1, 2$ .  $\frac{dy}{dx} = \frac{x^2+6x-1}{(x+3)^2}$

$m_T$  at  $(x=-1) = -\frac{3}{2}$ ;  $m_T$  at  $(x=2) = \frac{3}{5}$

$m_N$  at  $(x=-1) = \frac{2}{3}$ ;  $m_N$  at  $(x=2) = -\frac{5}{3}$   
(OR)

$$f'(x) = \frac{\cos x (4 - \cos x)}{(2 + \cos x)^2}$$

In I quadrant,  $\cos x, (4 - \cos x)$  &  $(2 + \cos x)^2$  are positive.  
 $\therefore f'(x) > 0$  in  $(0, \frac{\pi}{2}) \Rightarrow f(x)$  is  $\uparrow$  in  $\{0 < x < \frac{\pi}{2}\}$ .

- 8)  $n = 10, p = \frac{10}{100} = \frac{1}{10} \text{ & } q = \frac{9}{10}$ .

$$P(\text{at least one defective egg}) = P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - {}^{10}C_0 \left(\frac{q}{10}\right)^{10}$$

$$= 1 - \left(\frac{9}{10}\right)^{10}.$$

- 9) Let  $(a, 0)$  be the centre of any member of the family.  
 $\therefore (x-a)^2 + y^2 = a^2 \Rightarrow x^2 + y^2 = 2ax \rightarrow ①$

$$\Rightarrow 2x + 2y \cdot y' = 2a$$

$$\therefore \text{From } ①, x^2 + y^2 = (2x + 2yy')x \Rightarrow y' = \frac{y^2 - x^2}{2xy}.$$

- 10)  $[\vec{a} \vec{b} \vec{c}] = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{bmatrix} = -2(-12) + 2(12) + 4(-12)$   

$$= +24 + 24 - 48 = 0$$

$\therefore$  They are coplanar.

(2)

11)  $\vec{n} = \vec{n}_1 \times \vec{n}_2 = 7\hat{i} - 8\hat{j} - 3\hat{k}$

The required plane passes through the point  $(-1, 3, 2)$  & has  $\vec{n} = 7\hat{i} - 8\hat{j} - 3\hat{k}$ .  $\therefore$  Eqn  $\rightarrow 7(x+1) - 8(y-3) - 3(z-2) = 0$   
 $\Rightarrow 7x - 8y - 3z + 37 = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & -1 \end{vmatrix} = \hat{i}(-2+9) - \hat{j}(-1+9) + \hat{k}(3-6)$$

(OR)

Eqn of the reqd. plane is  $(3x-y+2z-4) + \lambda(x+y+z-2) = 0$   
 $\Rightarrow (3+\lambda)x + (-1+\lambda)y + (2+\lambda)z + (-4-2\lambda) = 0$   
It passes through  $(2, 2, 1)$   $\Rightarrow (3+\lambda)2 + (-1+\lambda)2 + (2+\lambda)1 + (-4-2\lambda) = 0$   
 $\Rightarrow 3\lambda + 2 = 0 \Rightarrow \lambda = -\frac{2}{3}$   
 $\therefore$  Eqn of the plane is  $7x - 5y + 4z - 8 = 0$ .

12)  $x : 0, 1, 2, 3$  (no. of heads)  
 $P(x=0) = \frac{1}{8}, P(x=1) = \frac{3}{8}, P(x=2) = \frac{3}{8}, P(x=3) = \frac{1}{8}$   
Mean =  $\sum x_i P(x_i) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{3}{2} = 1.5$   
Var( $x$ ) =  $\sum x_i^2 P(x_i) - \mu^2 = (0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}) - \frac{9}{4} = \frac{6}{8} = 0.75$

## SECTION-C

13)  $\tan^{-1} \left[ \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] = \tan^{-1} \left[ \frac{(\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta})}{(\sqrt{1+\cos\theta}) - (\sqrt{1-\cos\theta})} \right]$

Let  
 $x^2 = \cos\theta$   
 $\Rightarrow \theta = \cos^{-1} x^2$

$= \tan^{-1} \left[ \frac{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2} \right]$

$= \tan^{-1} \left[ \frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2} \right] = \tan^{-1} \left[ \frac{1 + \tan\theta/2}{1 - \tan\theta/2} \right]$

$= \tan^{-1} \left[ \frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{\theta}{2}} \right]$

$= \tan^{-1} \left[ \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) \right]$

$= \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$

14) Let  $\vec{d} = \lambda(\vec{a} \times \vec{b})$

$\therefore \vec{d} = \lambda(32\hat{i} - \hat{j} - 14\hat{k})$  &  $\vec{c} \cdot \vec{d} = 18$

$\Rightarrow \lambda(64 + 1 - 56) = 18 \Rightarrow \lambda = 2$ .

$\therefore \vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$

$$16) \frac{dy}{dx} = -\left(\frac{3xy+y^2}{x^2+xy}\right)$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = -\left(\frac{3x^2v+x^2v^2}{x^2+x^2v}\right)$$

$$\Rightarrow x \frac{dv}{dx} = \frac{-3v-v^2}{1+v} - v = \frac{-2v^2-4v}{1+v}$$

$$\Rightarrow \int \frac{v+1}{2v^2+4v} dv = \int -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{4} \log(2v^2+4v) + \log x = \log C$$

$$\Rightarrow x(2v^2+4v)^{1/4} = C \Rightarrow x^4(2v^2+4v) = C^4$$

$$\Rightarrow x^4 \left[ \frac{2y^2}{x^2} + \frac{4y}{x} \right] = C \quad (\text{Take } C^4 = C)$$

$$\Rightarrow 2y^2x^2 + 4yx^3 = C \Rightarrow \underline{\underline{C=6}} \quad (\text{when } x=1, y=1)$$

$$\therefore y^2x^2 + 2yx^3 = 3$$

$$17) \frac{3x^2+4x+5}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$\Rightarrow 3x^2+4x+5 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$$

Solving we get,  $A = -2$ ,  $B = 3/5$  &  $C = 22/5$ .

$$\begin{aligned} \therefore \int \frac{3x^2+4x+5}{(x-1)(x+2)(x-3)} dx &= -2 \int \frac{1}{x-1} dx + \frac{3}{5} \int \frac{1}{x+2} dx + \frac{22}{5} \int \frac{1}{x-3} dx \\ &= -2 \log|x-1| + \frac{3}{5} \log|x+2| + \frac{22}{5} \log|x-3| \end{aligned}$$

$$(15) \begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix}$$

$$R_1 \rightarrow R_1 - x^2 R_2$$

$$= \begin{vmatrix} a(1-x^4) & c(1-x^4) & p(1-x^4) \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix}$$

$$= (1-x^4) \begin{vmatrix} a & c & p \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix}$$

$$R_2 \rightarrow R_2 - x^2 R_1$$

$$= (1-x^4) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

$$R_1 \leftrightarrow R_2$$

$$= (x^4-1) \begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix} \quad (\text{OR})$$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 - R_2, \quad R_2 \rightarrow R_2 - R_3$$

$$\begin{vmatrix} 2 & 6 & 12 \\ 4 & 18 & 48 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

$$C_2 \rightarrow C_2 - 3C_1 \quad \& \quad C_3 \rightarrow C_3 - 6C_1$$

$$\begin{vmatrix} 2 & 0 & 0 \\ 4 & 6 & 24 \\ x-8 & -x-3 & -3x-16 \end{vmatrix} = 4 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 12 \\ x-8 & -x-3 & -3x-16 \end{vmatrix} = 0$$

~~OR~~

$$= 4 \left[ -3(3x+16) + 12(x+3) \right] = 0$$

$$\Rightarrow \underline{x = 4}$$

18)  $f(a) = 10$ ;  $LHL = 6a - 2b$ ;  $RHL = 2a + b$

$$\therefore 6a - 2b = 10 \text{ and } 2a + b = 10.$$

Solving we get  $a=3$  and  $b=4$ .

19)  $\cos^2 x \left( \frac{dy}{dx} \right) + y = \tan x$

$$\Rightarrow \frac{dy}{dx} + (\sec^2 x)y = \sec^2 x \tan x \quad (\text{L.D.E})$$

$$\therefore I.F = e^{\int \sec^2 x dx} = e^{\tan x}$$

$$\therefore y e^{\tan x} = \int (\tan x \cdot \sec^2 x) e^{\tan x} dx$$

$$\text{Let } \tan x = t$$

$$\therefore \sec^2 x dx = dt$$

$$\Rightarrow y e^{\tan x} = \int t \cdot e^t dt$$

On integrating by parts, we get,

$$y e^{\tan x} = t e^t - e^t + C = (\tan x) e^{\tan x} - e^{\tan x} + C$$

$$\therefore y = (\tan x - 1) + C e^{\tan x}$$

(OR)

$$y e^y dx = (y^3 + 2x e^y) dy$$

$$\Rightarrow \frac{dx}{dy} + \left(-\frac{2}{y}\right)x = y^2 e^{-y} \quad (\text{L.D.E})$$

$$\therefore I.F = e^{\int \frac{2}{y} dy} = y^2$$

$$\therefore x \cdot \frac{1}{y^2} = \int y^2 e^{-y} \cdot \frac{1}{y^2} dy = \int e^{-y} dy = -e^{-y} + C$$

$$\Rightarrow x = C y^2 - y^2 e^{-y}.$$

20)  $I = \int_1^3 (3x^2 + 1) dx$

Here  $a=1$ ,  $b=3$ ,  $nh=2$

$$I = \lim_{h \rightarrow 0} h \sum_{r=1}^n \left\{ 3[a + (r-1)h]^2 + 1 \right\}$$

$$= \lim_{h \rightarrow 0} h \left\{ 3[n^2 + h^2(1^2 + 2^2 + 3^2 + \dots + (n-1)^2) + 2ah(1+2+\dots+n-1)] + nh^2 \right\}$$

$$= \lim_{h \rightarrow 0} \left[ 3nh + 3h^3 \left( \frac{n(n-1)(2n-1)}{6} \right) + 6h^2 \left( \frac{n(n-1)}{2} \right) + nh \right]$$

$$= 3(2) + \frac{3(2)(2-0)(4-0)}{6} + \frac{6(2)(2-0)}{2} + 2$$

$$= 6 + 8 + 12 + 2 = \underline{\underline{28}}$$

21)  $P(E_1) = \frac{160}{400}$ ,  $P(E_2) = \frac{100}{400}$ ,  $P(E_3) = \frac{140}{400}$ , where

$E_1$ : selected person is a Smoker & non-Veg

$E_2$ : " " " Smoker & Veg

$E_3$ : " " " non-Smoker & Veg.

A: Person suffers from a chest disease.

$$P(A|E_1) = \frac{35}{100}, P(A|E_2) = \frac{20}{100}, P(A|E_3) = \frac{10}{100}, P(E_1|A) = ?$$

$$\begin{aligned} \therefore P(E_1|A) &= \frac{\frac{160}{400} \times \frac{35}{100}}{\frac{160}{400} \times \frac{35}{100} + \frac{100}{400} \times \frac{20}{100} + \frac{140}{400} \times \frac{10}{100}} = \frac{160 \times 35}{160 \times 35 + 100 \times 20 + 140 \times 10} \\ &= \frac{112}{112+40+28} = \frac{112}{180} = \frac{28}{45} // \end{aligned}$$

22) Let  $u = x^{\tan x}$

$$\Rightarrow \log u = \tan x \log x$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \sec^2 x \log x + \frac{\tan x}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{\tan x} \left[ \sec^2 x \log x + \frac{\tan x}{x} \right]$$

$$\therefore \frac{d^2u}{dx^2} = x^{\tan x} \left[ \sec^2 x \log x + \frac{\tan x}{x} \right] + \underline{\underline{(x^{\tan x})^x \left[ \log x + \frac{x \sec^2 x}{\tan x} \right]}}$$

$$\text{Let } v = (\tan x)^x$$

$$\Rightarrow \log v = x \log \tan x$$

$$\frac{1}{v} \cdot \frac{dv}{dx} = \log \tan x + \frac{x \sec^2 x}{\tan x}$$

$$\therefore \frac{dv}{dx} = (\tan x)^x \left[ \log \tan x + \frac{x \sec^2 x}{\tan x} \right]$$

$$\underline{\underline{(x^{\tan x})^x \left[ \log x + \frac{x \sec^2 x}{\tan x} \right]}}$$

23)  $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx$

$$\sqrt{\tan x} + \sqrt{\cot x} = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$$

$$= \sqrt{2} \int_{-1}^1 \frac{dt}{\sqrt{1-t^2}}$$

$$= \sqrt{2} \left[ \sin^{-1} t \right]_{-1}^1$$

$$= \sqrt{2} [\sin^{-1}(1) - \sin^{-1}(-1)] = \sqrt{2} [\sin^{-1} 1 + \sin^{-1} 1]$$

$$= \sqrt{2} \left[ \frac{\pi}{2} + \frac{\pi}{2} \right] = \underline{\underline{\sqrt{2}\pi}}$$

(OR)

$$I = \int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x} \rightarrow ①$$

$$\text{Let } \sin x - \cos x = t$$

$$\therefore (\cos x + \sin x) dx = dt$$

$$(\sin x - \cos x)^2 = t^2$$

$$\Rightarrow 1 - \sin 2x = t^2$$

$$\Rightarrow \sin x \cos x = \frac{1-t^2}{2}$$

$$x=0 \Rightarrow t=-1$$

$$x=\pi/2 \Rightarrow t=1$$

$$I = \int_0^{\pi} \frac{(\pi-x) dx}{a^2 \cos^2(\pi-x) + b^2 \sin^2(\pi-x)}$$

Applying  $\int_a^a f(x) dx = \int_0^a f(a-x) dx$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x) dx}{a^2 \cos^2 x + b^2 \sin^2 x} \rightarrow ②$$

$$\text{Adding } ① \& ②, 2I = \pi \int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$\Rightarrow 2I = 2\pi \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

Applying  $\int_0^{2a} f(x) dx = 2 \int_0^a f(2a-x) dx$   
if  $f(2a-x) = f(x)$

$$= \pi \int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2 \tan^2 x}$$

÷ Num. & Den. by  $\cos^2 x$ .

$$\text{let } \tan x = t$$

$$\Rightarrow \sec^2 x dx = dt$$

$$x=0 \Rightarrow t=0$$

$$x=\pi/2 \Rightarrow t=\infty$$

$$= \pi \int \frac{dt}{a^2 + b^2 t^2}$$

$$= \frac{\pi}{ab} \left[ \tan^{-1} \frac{bt}{a} \right]_0^\infty = \frac{\pi}{ab} [\tan^{-1} \infty - \tan^{-1} 0]$$

$$= \frac{\pi}{ab} \left[ \frac{\pi}{2} - 0 \right] = \frac{\pi^2}{2ab} //$$

#### SECTION-D :-

$$24) \text{ Line } ① \text{ is } \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda + 2, z = 4\lambda + 3 \rightarrow (a)$$

$$\text{This point lies on the line } ②: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$$

$$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = 4\lambda+3$$

$$\Rightarrow \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2} = 4\lambda+3 \Rightarrow \underline{\underline{\lambda = -1}}$$

$$\frac{3(-1)+1}{2} = 4(-1)+3 \\ -1 = -1$$

To get point of intersection, put  $\lambda = -1$  in (a)

$$\therefore x = -1, y = -1, z = -1$$

$$25) * : x \rightarrow x, \text{ where } x = R - \{-1\}, \text{ defined as } x * y = x + y + xy$$

(i) Let  $x, y \in R - \{-1\}$

$$\begin{aligned} x * y &= x + y + xy \\ &= y + x + yx \\ &= y * x \end{aligned}$$

$\Rightarrow x * y = y * x \Rightarrow *$  operation is commutative.

(ii) Let  $x, y, z \in R - \{-1\}$

$$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy)z \\ = x + y + z + xy + yz + zx + xyz$$

$$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x(y + z + yz) \\ = x + y + z + xy + yz + zx + xyz$$

$\Rightarrow *$  operation is associative.

(iii) Let  $e \in R - \{-1\}$  be identity element of  $a \in R - \{-1\}$

$$\therefore a * e = e * a = a$$

$$\Rightarrow a + e + ae = a \text{ and } e + a + ea = a$$

$$\Rightarrow e(1+a) = 0 \text{ and } e(a+1) = 0$$

$$\Rightarrow e = 0 \in R - \{-1\}.$$

(iv) Let  $b \in R - \{-1\}$  be the inverse of  $a \in R - \{-1\}$

$$\therefore a * b = e \text{ and } b * a = e$$

$$\Rightarrow a + b + ab = 0 \text{ & } b + a + ba = 0$$

$$\Rightarrow b(a+1) = -a \Rightarrow b = \frac{-a}{a+1} (a \neq -1)$$

(OR)

(i) Let  $(e_1, e_2) \in A$  be the identity element of  $*$ .

$$\therefore (e_1, e_2) * (a, b) = (a, b) \text{ for } (a, b) \in A.$$

$$\Rightarrow (e_1a, e_2 + e_1b) = (a, b) \Rightarrow e_1 = 1 \text{ and } e_2 + 1 \times b = b \Rightarrow e_2 = 0.$$

$\therefore$  Identity element is  $(1, 0)$ .

(ii) Let  $(c, d)$  be the inverse of  $(a, b)$

$$\therefore (a, b) * (c, d) = (e_1, e_2)$$

$$\Rightarrow (ac, b + ad) = (1, 0) \Rightarrow ac = 1, b + ad = 0 \\ \Rightarrow c = \frac{1}{a}, d = -\frac{b}{a}$$

$\therefore \left(\frac{1}{a}, -\frac{b}{a}\right)$  is the inverse of  $(a, b)$ .

(iii) Inverse of  $(5, 3) = \left(\frac{1}{5}, -\frac{3}{5}\right)$  and

Inverse of  $(\frac{1}{2}, 4) = (2, -8)$ .

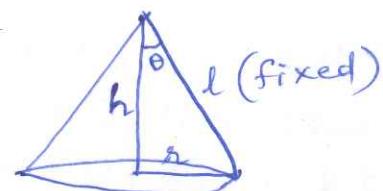
26) Let  $l, h, V$  be the slant height, height & volume of the cone.

$$\therefore r = \sqrt{l^2 - h^2}$$

$$Vol = \frac{1}{3} \pi (l^2 - h^2) h = \frac{1}{3} \pi (l^2 h - h^3)$$

$$\frac{dV}{dh} = \frac{\pi}{3} (l^2 - 3h^2), \quad \frac{dV}{dh} = 0 \Rightarrow l^2 = 3h^2 \Rightarrow h = \frac{l}{\sqrt{3}}$$

If  $\theta$  is the semi-vertical angle,  $\tan \theta = \frac{r}{h} = \frac{\sqrt{l^2 - h^2}}{h} = \sqrt{2}$ .  
Now  $\frac{d^2 V}{dh^2} = -6h < 0 \Rightarrow Vol$  is max.



27)  $|A| = 30$

$$\text{Adj } A = \begin{bmatrix} 4 & 2 & -10 \\ 1 & 8 & 5 \\ 7 & -4 & 5 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{30} \begin{bmatrix} 4 & 2 & -10 \\ 1 & 8 & 5 \\ 7 & -4 & 5 \end{bmatrix}$$

$$\therefore X = \frac{1}{30} \begin{bmatrix} 4 & 2 & -10 \\ 1 & 8 & 5 \\ 7 & -4 & 5 \end{bmatrix} \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix} \quad \therefore x = -1 \\ y = 2 \\ z = 3$$

(OR)

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

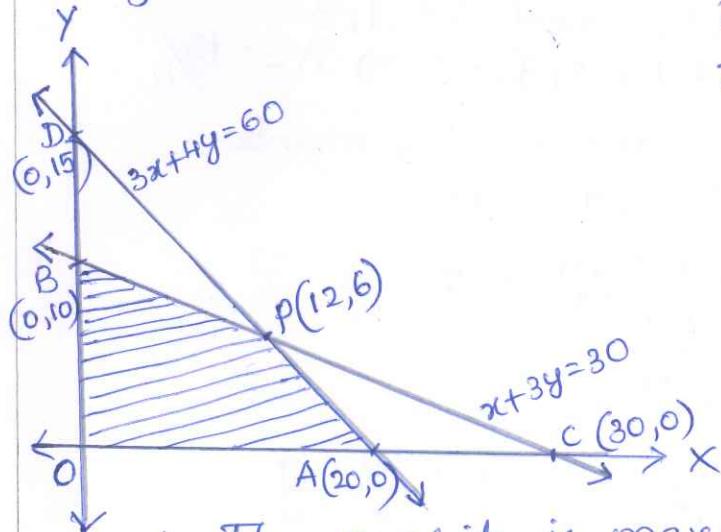
28) Let  $x$  &  $y$  be the no. of teaching aids of type A & B respectively.

$$\text{Maximize } Z = 80x + 120y$$

Subject to constraints  $9x + 12y < 180 \rightarrow ①$

$$x + 3y < 30 \rightarrow ②$$

$$x \geq 0, y \geq 0 \rightarrow ③$$



corner points	$Z = 80x + 120y$
O(0,0)	0
A(20,0)	1600
P(12,6)	1680 $\rightarrow$ Max
B(0,10)	1200

$\therefore$  The profit is max. at P(12,6) when 12 aids of type A & 6 aids of type B are made.

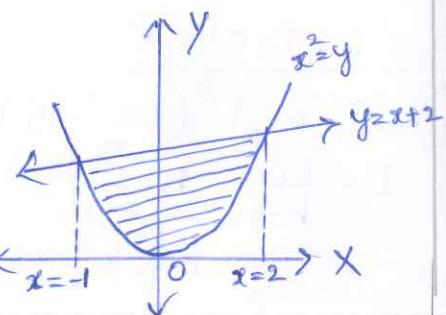
Max. Profit = ₹ 1680 per week.

29)  $x^2 = y$  and  $y = x+2$ . Solving we get  $x = -1, x = 2$ .

$$\text{Reqd area} = \int_{-1}^2 (x+2) dx - \int_{-1}^2 x^2 dx$$

$$= \left[ \frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2$$

$$= \left( 2 + 4 - \frac{8}{3} \right) - \left( \frac{1}{2} - 2 + \frac{1}{3} \right) = \underline{\underline{\frac{27}{6}} \text{ sq. units}}$$

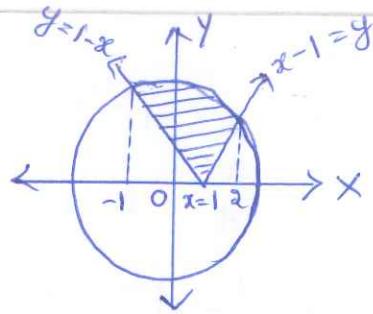


(OR)

$$y = |x-1| \text{ and } y = \sqrt{5-x^2}$$

$$y = \begin{cases} (x-1) & \text{if } x \geq 1 \\ (1-x) & \text{if } x < 1 \end{cases} \quad \& \quad y^2 + x^2 = 5$$

Solving  $y = \sqrt{5-x^2}$  and  $y = x-1$   
we get  $x = -1, x = 2$ .



$$\begin{aligned} \therefore \text{Req. area} &= \int_{-1}^2 \sqrt{5-x^2} dx + \int_{-1}^1 (x-1) dx - \int_{-1}^2 (x-1) dx \\ &= \left[ \frac{5}{2} \sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{x}{2} \sqrt{5-x^2} \right]_{-1}^2 + \frac{1}{2} [(x-1)^2]_{-1}^1 - \frac{1}{2} [(x-1)^2]_{-1}^2 \\ &= \left( \frac{5\pi}{4} - \frac{1}{2} \right) \text{ sq. units.} \end{aligned}$$

SET-BSECTION-A:

2)  $\frac{6}{7}$

4)  $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

SECTION-B

5) d<sub>r</sub>'s of the 2 lines are 4,  $3\lambda$ , 5 and  $2\lambda$ , 1, 3.  
Since they are  $\perp^r$ ,  $4(2\lambda) + 3\lambda \cdot 1 + 5(3) = 0 \Rightarrow \lambda = -\frac{15}{11}$ .

12) Let (0, a) be the centre of any family member.

$$\therefore x^2 + (y-a)^2 = a^2 \Rightarrow x^2 + y^2 = 2ay \rightarrow ①$$

$$\Rightarrow 2x + 2yy' = 2ay' \Rightarrow \frac{2x + 2yy'}{y'} = 2a$$

$$\text{From ①, } x^2 + y^2 = \left( \frac{2x + 2yy'}{y'} \right) y$$

$$\Rightarrow y'(x^2 + y^2) = 2xy + 2y^2y' \Rightarrow y' = \frac{2xy}{x^2 - y^2}$$

10)  $X \rightarrow 0, 1, 2, 3$

$$\text{Mean} = \frac{3}{2} = 1.5, \quad \text{Var}(x) = \frac{3}{4} = 0.75$$

SECTION-C

14)  $I = \int_1^3 (3x^2 - 5) dx$ . Here  $a=1, b=3, nh=2$

$$I = \lim_{h \rightarrow 0} h \sum_{n=1}^{\frac{n}{h}} \{3[(a+(n-1)h)]^2 - 5\}$$

$$= \lim_{h \rightarrow 0} h \left\{ 3 \left[ na^2 + h^2 (1^2 + 2^2 + \dots + (n-1)^2) + 2ah(1+2+\dots+n-1) \right] - 5n \right\}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \left[ 3nh + 3 \frac{nh(nh-h)(2nh-h)}{6} + 6 \frac{nh(nh-h)}{2} - 5nh \right] \\
 &= 3(2) + \frac{3(2)(2-0)(4-0)}{6} + \frac{6(2)(2-0)}{2} - 5(2) \\
 &= 6 + 8 + 12 - 10 = \underline{\underline{16}}
 \end{aligned}$$

16) Let  $\vec{d} = \lambda (\vec{a} \times \vec{b})$

$$\begin{aligned}
 \therefore \vec{d} &= 32\lambda \hat{i} - 14\lambda \hat{j} - 14\lambda \hat{k} \quad \& \vec{c} \cdot \vec{d} = 15 \\
 \Rightarrow \lambda(64+1-56) &= 15 \Rightarrow 9\lambda = 15 \Rightarrow \lambda = 5/3 \\
 \therefore \vec{d} &= 160\hat{i} - 5\hat{j} - 70\hat{k}
 \end{aligned}$$

17)  $\frac{2x^2+5x+7}{(x-2)(x-3)^2} = \frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$\Rightarrow 2x^2+5x+7 = A(x-3)^2 + B(x-2)(x-3) + C(x-2)$$

Solving we get,  $A = 25$ ,  $B = -23$ ,  $C = 40$ .

$$\begin{aligned}
 \therefore \int \frac{2x^2+5x+7}{(x-2)(x-3)^2} dx &= 25 \int \frac{dx}{x-2} - 23 \int \frac{dx}{x-3} + 40 \int \frac{dx}{(x-3)^2} \\
 &= 25 \log|x-2| - 23 \log|x-3| - \frac{40}{x-3} \underline{\underline{=}}
 \end{aligned}$$

26) Line ① is  $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-3}{-5} = \lambda$

$$\Rightarrow x = 2\lambda + 1, y = 3\lambda + 1, z = -5\lambda + 3$$

This point lies on line ②:  $\frac{x-3}{1} = \frac{y-4}{-3} = \frac{z+2}{2}$

$$\Rightarrow \frac{2\lambda+1-3}{1} = \frac{3\lambda+1-4}{-3} = \frac{-5\lambda+3+2}{2}$$

$$\Rightarrow 2\lambda-2 = \frac{3\lambda-3}{-3} = \frac{-5\lambda+5}{2}$$

$$\Rightarrow 2\lambda-2 = \frac{3\lambda-3}{-3} \Rightarrow -6\lambda+6 = 3\lambda-3 \Rightarrow -9\lambda = -9 \Rightarrow \lambda = 1$$

$$\Rightarrow 2\lambda-2 = \frac{3\lambda-3}{-3} \Rightarrow -6\lambda+6 = 3\lambda-3 \Rightarrow -9\lambda = -9 \Rightarrow \lambda = 1$$

$\therefore$  The point of intersection is  $(3, 4, -2)$

29) Let  $x$  &  $y$  be the no. of products A & B made per day.

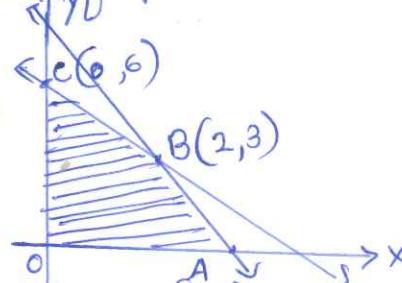
Maximise  $Z = 7x + 4y$

Subject to constraints

$$3x + 2y \leq 12 \rightarrow ①$$

$$3x + y \leq 9 \rightarrow ②$$

$$x \geq 0, y \geq 0 \rightarrow ③$$



for max profit, 2 units of A & 3 units of B.

corner points	$Z = 7x + 4y$
O(0,0)	0
A(3,0)	21
B(2,3)	26 → Max
C(0,6)	24

SET-C

$$17) \frac{x^2+5}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$\Rightarrow x^2+5 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

Solving we get,  $A = 2/3$ ,  $B = 1/3$ ,  $C = 3$

$$\therefore \int \frac{x^2+5}{(x-1)(x+2)^2} dx = \frac{2}{3} \int \frac{dx}{x-1} + \frac{1}{3} \int \frac{dx}{x+2} + 3 \int \frac{dx}{(x+2)^2}$$

$$= \frac{2}{3} \log|x-1| + \frac{1}{3} \log|x+2| - \frac{3}{(x+2)^2}$$

$$27) |A| = 67$$

$$A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\therefore X = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \quad \therefore x = 3 \\ y = -2 \\ z = 1.$$

$$29) \text{ Solving } 4y = 3x^2 \text{ and } 2y = 3x + 12, \text{ we get} \\ x = -2, x = 4$$

$$\text{Req. area} = \int_{-2}^4 \left( \frac{3x+12}{2} - \frac{3x^2}{4} \right) dx$$

$$= \frac{3}{4} \int_{-2}^4 8 + 2x - x^2$$

$$= \underline{\underline{27 \text{ sq. units}}}$$

